Constraining $\alpha_s(M_Z)$ from the Hidden Sector

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Abstract

By including the effects of superstring thresholds, we reconsider minimal string unification together with the requirement of producing a supersymmetry-breaking gluino condensate in the hidden sector. This gives, for examples of phenomenologically-viable Z_8' and other related orbifolds, the constraint that $0.1215 \leq \alpha_s(M_Z) \leq 0.1270$. In such models, a hidden photino can be a source of cosmological dark matter detectable by gravitational microlensing.

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Superstrings provide a hope for a mathematical framework underpinning both particle theory and quantum gravity, yet a testable prediction remains elusive. Predictions concerning quantum gravity are generally too small to be detectable but, of course, calculation of any parameter in the standard particle theory would be an adequate confirmation. We can, however, envisage a different empirical confrontation where the hidden sector of the superstring impacts on the visible sector and a connecting bridge at M_{string} could then provide consistency checks.

Most attempts at superstring phenomenology over the last ten years have been very speculative and, in any case, have concentrated on the link to grand unification in the visible sector. By this we mean that in the heterotic $E_8 \times E_8'$ superstring only, say, the first E_8 is involved. In this letter we use recent results for superstring thresholds to link the visible sector to the hidden sector (E_8') and discuss the implications of hidden sector supersymmetry breaking [1] for the value of the QCD coupling constant $\alpha_s(M_Z)$ and for cosmology. On the hidden side we necessarily follow a top-down approach because nothing is known empirically. The treatment of the visible sector is partially bottom-up since we input the low energy data.

In the visible sector, the standard model of particle theory is well established up to 100 GeV and a supersymmetric grand unification at $M_{GUT} \sim 2 \times 10^{16}$ GeV, corresponding to a supersymmetry breaking scale ~ 1 TeV, has been advocated [2–4] (see, however, Ref. [5]). In a superstring the relevant scale is denoted M_{string} which is related to Newton's gravitational constant (M_{Planck} or the reduced form $\overline{M}_{Planck} = M_{Planck}/\sqrt{8\pi}$) through the string coupling constant g_{string} in a calculable way. Here we shall assume an orbifold compactification with, below M_{string}, only the minimal supersymmetric standard model (MSSM) fields having standard $U(1)_Y$ normalization. Since M_{string} lies somewhat higher (typically M_{string} is a few times 10^{17} GeV) than M_{GUT}, a successful minimal string unification at the higher scale necessitates sizeable superstring threshold corrections.

The hidden sector is often assumed to generate supersymmetry breaking via a gluino condensate for $\alpha_{hidden}(\mu_0) \to \infty$ at $\mu_0 \sim 10^{13}$ GeV [6]. At the same time this may generate

Cold Dark Matter (CDM) and have consequences for inflation and structure formation in the visible sector. Big Bang Nucleosynthesis (BBN) constrains the massless degrees of freedom in the hidden sector and thereby, in principle, the possible breakings of the hidden E_8'; however, this will depend on the ratio (r) of the temperatures of the hidden to visible sectors.

In the heterotic superstring M_{string} is given by [7,8]

$$M_{string} = g_{string} \times 5.3 \times 10^{17} \text{ GeV} ,\tag{1}$$

where g_{string} is a dimensionless coupling constant related to the real part of the dilaton superfield expectation value by $g_{string}^{-2} = ReS$. Although, according to general arguments [9], one expects the superstring to be strongly coupled, one may also have the low energy string coupling satisfying $\alpha_{string} = g_{string}^2/4\pi < 1$ and hence a perturbative low energy theory. We shall see numerically that to obtain a reasonable supersymmetry breaking scale, $g_{string} \gtrsim 0.2$, so M_{string} must lie above the old GUT scale (and becomes the new, higher, effective GUT scale) while lying safely below $\overline{M}_{Planck}$.

The renormalization group equations for the $SU(3) \times SU(2) \times U_Y(1)$ gauge couplings α_a $(a = 1, 2, 3)$ in the visible sector of the superstring have the form for $\mu \leq M_{string}$:

$$\frac{1}{\alpha_a(\mu)} = \frac{k_a}{\alpha_{string}} + \frac{b_a}{4\pi} \ln \frac{M_{string}^2}{\mu^2} + \frac{1}{4\pi}\Delta_a ,\tag{2}$$

where $k_1 = \frac{5}{3}, k_2 = k_3 = 1$. Here b_a are the one-loop renormalization group β function coefficients for the gauge groups and Δ_a represent the corresponding superstring thresholds [10,11]. In orbifold models, $\Delta_a = -\sum_i b_a^{'(i)}\Delta^{(i)}$ where $\Delta^{(i)}$ are threshold factors depending on the (1,1) string moduli for the orbifold subplanes $i = 1, 2, 3$ and $b_a^{'(i)}$ are related to the β function coefficients of the N = 2 sector of the orbifold. The latter are given by [12,13]

$$b_a^{'(i)} = -C(G_a) + \sum_{R_a} T(R_a)(1 + 2n_{R_a}^{(i)}) ,\tag{3}$$

while, summing over the three subplanes gives

$$b_a' = -3C(G_a) + \sum_{R_a} T(R_a)(3 + 2n_{R_a}) ,\tag{4}$$

in which $T(R_a)$ are the Dynkin indices and n_{R_a} are the modular weights of the light matter fields in irreducible representations R_a. If the (1,1) moduli T^i are *either* all equal $T = T^1 = T^2 = T^3$ (isotropic orbifold) *or* one of them is by far the largest $T = T^1 \gg T^2, T^3$ (squeezed orbifold) then we may write $\Delta_a = b'_a \Delta$ where $b'_a = \sum_i b'^{(i)}_a$ for the former case and $b'_a = b'^{(1)}_a$ for the latter case. In either case, Δ is a common function whose explicit form in the large T limit is given for simple cases by [10]

$$\Delta = \ln(|T + \overline{T}||\eta(T)|^4) \tag{5}$$

where η is the Dedekind function.

Elimination of α_{string} in Eqs. (2) gives the generalization of the GQW equations [14]:

$$\sin^2 \theta_W(M_Z) = \frac{k_2}{k_1 + k_2} - \frac{k_1}{k_1 + k_2} \frac{\alpha_{em}(M_Z)}{4\pi} \left[A \ln \left(\frac{M^2_{string}}{M^2_Z} \right) - A' \Delta \right], \tag{6}$$

$$\alpha_s^{-1}(M_Z) = \frac{k_3}{k_1 + k_2} \left[\alpha_{em}^{-1}(M_Z) - \frac{1}{4\pi} B \ln \left(\frac{M^2_{string}}{M^2_Z} \right) + \frac{1}{4\pi} B' \Delta \right], \tag{7}$$

where $A = \frac{k_2}{k_1} b_1 - b_2$, $B = b_1 + b_2 - \frac{k_1 + k_2}{k_3} b_3$ and A', B' are obtained from A, B by the substitution $b_a \to b'_a$. For the MSSM one has the values $A = \frac{28}{5}, B = 20$.

The renormalization group equation for the coupling constant in the hidden sector corresponding to any subgroup G' of E'_8 is

$$\frac{1}{\alpha_{G'}(\mu)} = \frac{1}{\alpha_{string}} + \frac{b_{G'}}{4\pi} \ln \frac{M^2_{string}}{\mu^2} \tag{8}$$

Here we have assumed no matter fields are present in the hidden sector, so the corresponding superstring thresholds vanish. The value of $\mu = \mu_0$ where $\alpha_{G'}(\mu_0) \to \infty$ is thence given by:

$$\mu_0 = M_{string} \exp \left[\frac{2\pi}{\alpha_{string} b_{G'}} \right]. \tag{9}$$

The scale μ_0 is determined by assuming the gravitino mass arises from the standard effective supergravity coupling between the gravitino and the gaugino bilinear ($\langle \chi\chi \rangle \sim \mu_0^3$). This gives a gravitino mass (within an order of magnitude)

$$m_{3/2} \sim \mu_0^3 / \overline{M}^2_{Planck} = 100 \text{ GeV} - 1 \text{ TeV} , \tag{10}$$

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to obtain the correct visible sector supersymmetry breaking. We will consider values in the range $\mu_0 = (0.4-4) \times 10^{13}$ GeV taking into account the above range and order of magnitude factors. To obtain the hidden sector gaugino condensate at $e.g.$ $\mu_0 \sim 10^{13}$ GeV (corresponding to visible supersymmetry breaking at ~ 1 TeV) we consider the quadratic Casimir, $C_2(G') = -\frac{1}{3}b_{G'}$, ranging from 2 (for SU(2)) to 30 (for E(8)). Recall that $C_2(G')$ = N for SU(N) and 12,18,30 for $E_{6,7,8}$ respectively. The results of solving Eq. (9) for M_{string} are displayed in Fig. 1 for the above range of μ_0.

Given M_{string} from the hidden sector analysis and a value of $\gamma = B'/A'$, we can now determine from Eqs. (6,7) the value of $A'\Delta$ and $\alpha_s(M_Z)$ respectively, for the range of allowed empirical values for $\sin^2\theta_W(M_Z)$ and $\alpha_{em}(M_Z)$ which, taking into account low energy MSSM, are currently [15]

$$\sin^2\theta_W(M_Z) = 0.2313 \pm 0.0003$$

$$\alpha_{em}^{-1}(M_Z) = 128.09 \pm 0.09 . \tag{11}$$

Since Eqs. (6,7) determine $\alpha_s(M_Z)$ only in terms of the ratio γ, we must try to determine γ. As a first example we use the compactification of the heterotic string on the Z'_8 orbifold, as discussed in [12,13]. In that particular example, for the three complex planes corresponding to the three two-dimensional subtori within the orbifold, the MSSM states were assigned modular weights as follows: $n_{Q_{1,2,3}} = (0,-1,0); n_{D_{1,2,3}} = (-1,0,0); n_{U_1} = (0,-\frac{1}{2},-\frac{1}{2}); n_{U_{2,3}} = (-\frac{3}{4},-\frac{15}{8},-\frac{3}{8}); n_{L_1} = (-\frac{14}{8},-\frac{3}{8},-\frac{7}{8}); n_{L_{2,3}} = (-\frac{14}{8},-\frac{7}{8},-\frac{3}{8}); n_{E_1} = (-1,0,0); n_{E_{2,3}} = (-\frac{3}{4},-\frac{15}{8},-\frac{3}{8}); n_H = (-\frac{1}{2},-\frac{3}{4},-\frac{3}{4}); n_{\overline{H}} = (-\frac{14}{8},-\frac{3}{8},-\frac{7}{8})$. Using the definitions given above for the separate orbifold subplanes $i = 1,2,3$, this gives $b_1'^{(1,2,3)} = -\frac{15}{2}, -\frac{145}{2}, \frac{35}{2}$ yielding $b_1' = -\frac{50}{3}$; $b_2'^{(1,2,3)} = -\frac{5}{2}, -\frac{29}{4}, \frac{7}{4}$ yielding $b_2' = -8$; and finally $b_3'^{(1,2,3)} = -\frac{3}{2}, -\frac{29}{4}, \frac{7}{4}$ yielding $b_3' = -7$. In this case, the values of A', B' defined above are $A' = -2$ and $B' = -6$ giving $\gamma = 3$. For the other viable orbifolds cited in [13] the results are similar. For possible assignments of modular weights in squeezed Z_6 and $Z_2 \times Z_2$ orbifold models one can find $\gamma = 3$, just as in the Z'_8 case.

Of course, a range of values for γ is possible for a specific construction. But for all these

special choices of modular weights, one has $\gamma = 3$ so in Fig. 2 we show the relationship between $C_2(G')$ and $\alpha_s(M_Z)$ for this value of γ and taking $\mu_0 = 10^{13}$ GeV. Curves are shown for the central values of $\alpha_{em}(M_Z)$ and $\sin^2 \theta_W(M_Z)$ and for the extremes of the allowed ranges. Note that $\alpha_s(M_Z)$ is maximized for $\alpha_{em}(M_Z)$ maximized and $\sin^2 \theta_W(M_Z)$ minimized, and *vice versa*. The shifted short(long) dashed curves in Fig. 2 correspond to $\mu_0 = 0.4(4.0) \times 10^{13}$ GeV, showing that the results are relatively insensitive to μ_0. The range of $\alpha_s(M_Z)$, if we assume $C_2(\mathrm{SU}(2)) = 2 \le C_2(G') \le C_2(\mathrm{E}_7) = 18$, to allow the CDM hidden photino discussed below, is seen from Fig. 2 (for $\mu_0 = 10^{13}$ GeV) to be

$$0.1215 \le \alpha_s(M_Z) \le 0.1270 \ .$$

This is to be compared to the allowed LEP range $\alpha_s(M_Z) = 0.121 \pm 0.005$ [15].

In the above, we have taken the particular value $\gamma = 3$ which occurs in three simple examples given in [13] seriously. If we instead regard γ as an arbitrary rational number and input $\mu_0 = 10^{13}$ GeV, the full allowed ranges of $\alpha_{em}(M_Z), \sin^2 \theta_W(M_Z)$ and $\alpha_s(M_Z) = 0.121 \pm 0.005$ [15] then we find $2.62(2.5) \le \gamma \le 3.73(3.75)$ for $2 \le C_2(G') \le 18(30)$. This is illustrated in Fig. 3 where the three cases $\gamma = 2.62, 3.00$ and 3.73 are shown with uncertainties as in Fig. 2. Thus Fig. 3 provides constraints on γ and $C_2(G')$ for orbifold constructions to be consistent with LEP data, the gaugino condensate idea and the hidden photino CDM candidate disussed below. In any specific orbifold compactification, the quantities γ and $C_2(G')$ may be determined (or chosen from a range consistent with modular invariance) and therefore $\alpha_s(M_Z)$ is fixed up to a narrow range coming from uncertainties in (11) and an allowed range of μ_0. In principle, one should do a full two loop analysis and include the effects of a detailed MSSM spectrum in the form of low energy threshold corrections. However, given the uncertainties at M_{string}, these are less important in this type of analysis.

All these results depend on being able to obtain the necessary value for the moduli-dependent superstring threshold correction Δ determined by Eqs. (6,7) in a given orbifold construction. For the combination $A'\Delta$, the solution is mostly sensitive to $C_2(G')$, or equivalently M_{string}. For example, for $2 \le C_2(G') \le 18$ and $\mu_0 = 10^{13}$ GeV we find $38 \ge A'\Delta \ge 25$,

independently of γ. Given the value of A', there may be naturalness constraints on the size of $|\Delta|$ coming from the values of the moduli VEVs by which it is determined, so one might be restricted to large hidden sector gauge groups. However, as shown in [16], it is possible to obtain sufficiently large $|\Delta|$ in a Z_8' orbifold with $A' = -2$ for natural values of the moduli VEVs by including continuous Wilson lines; the latter are, in any case, generally necessary for the required symmetry breaking pattern in both the visible and hidden sectors.

Now we turn to the cosmological aspects and ramifications for the hidden sector. From the above, an illustrative scenario is where $g_{string} \sim 0.7$ and the condensate occurs in an SU(5) gauged subgroup of E_8'; for example, the breaking of E_8' by Wilson lines could give the rank 8 subgroup SU(5) × SU(4) × U(1). The SU(4) condensates are sufficiently heavy to decay gravitationally before BBN [17] and the hidden photino associated with the U(1) will have a mass comparable to the visible supersymmetry breaking scale of ~ 1 TeV. Let us now pursue the general idea that such a shadow photino is the origin of some, or all, cosmological dark matter.

A serious constraint on the number of hidden sector massless degrees of freedom arises from the agreement of the visible standard model with BBN. The hidden sector photons change the effective number of degrees of freedom according to:

$$g_{eff}^* = g_{visible}^* + g_{hidden}^* \left(\frac{T_{hidden}}{T_\gamma}\right)^4 \tag{12}$$

where $g_{visible}^*$ is the visible sector degrees of freedom at nucleosynthesis (10.75 for $e^\pm(3.5)$, $\nu_i(5.25)$ and $\gamma(2.0)$) while $g_{hidden}^* = 2N_{\gamma,hidden}$ for $N_{\gamma,hidden}$ hidden photons. The upper limit on g_{eff}^* is ~ 11.45 [18] and so there cannot be any shadow photon unless $T_{hidden} < T_\gamma$; if $T_{hidden} = T_\gamma$, no such extra massless state is permissible. (Note that in the older literature [19] the upper limit on g_{eff}^* was weaker: $g_{eff}^* < 13$).

As far as Eq. (12) and BBN are concerned, a key issue is the ratio of hidden to visible temperatures at the BBN era. If we adopt a hidden photino mass in the range 100 GeV - 1 TeV, and assume that it forms all the dark matter ($\Omega_{\tilde\gamma} \sim 1$), then the temperature ratio $(T_{hidden}/T_\gamma) \sim 10^{-3.3}$ to $10^{-3.7}$, since the number density of hidden photinos $n_{\tilde\gamma,hidden} \propto$

T^3_{hidden} just as $n_\gamma \propto T^3_\gamma$ and we assume $n_{\tilde{\gamma},hidden} = \frac{3}{4}n_\gamma$ at a very early era when $T = T_\gamma = T_{hidden}$ close to $T = M_{Planck}$.

Consider a model with normal sector gauge group G and hidden sector gauge group G_{hidden}, whose origins are both in our heterotic superstring theory. The symmetry breaking patterns, and herefore the phase transition structure of such a model, can be quite complicated, and the temperatures T_γ and T_{hidden} of the two sectors can evolve very differently, but are typically intertwined by inflation.

As an example, consider a scenario in which there are two inflationary epochs. Suppose that an inflaton, *e.g.* the dilaton, induces a period of inflation affecting equally the visible and hidden sectors. The universe supercools exponentially since the scale factor increases exponentially and $RT =$ constant. Most of the required e-foldings may be accomplished during this inflationary phase. The final, say, ~ 8 e-foldings may arise from an inflation induced, for example, by a superstring modulus field which couples to the visible matter and not to the hidden sector. This second inflationary phase can occur at the weak scale as suggested by Ref. [20] to solve the cosmological moduli problem. Reheating can then occur in the visible sector back to approximately the critical temperature, while no reheating is possible in the hidden sector. This leads to a temperature ratio between hidden and visible sectors $e^{-8} \sim 10^{-3.5}$ as required. This is just one possible scenario to illustrate how enough dark matter could be generated to make $\Omega = 1$.

Finally, we come to the testability of our dark matter proposal. The hidden photinos have a Jeans mass, M_{Jeans}, given by $M_{Jeans} = M^3_{Planck}/(m_{\tilde{\gamma},hidden})^2 \sim 10^{51}$ GeV $\sim 10^{-6}M_\odot$ for $m_{\tilde{\gamma},hidden} = 1$ TeV; for the lower value $m_{\tilde{\gamma},hidden} = 100$ GeV, $M_{Jeans} \sim 10^{-4}M_\odot$. Since this is CDM we expect it to have clumped gravitationally at this scale and so our galactic halo will be comprised of these hidden objects. The accretion of hidden photinos into such MACHOs will cause gravitational microlensing of distant stars, and detection of the resultant temporary achromatic amplification is a possibility. The method [21,22] is practical for dark matter objects in the range $10^{-7}M_\odot$ to $10^2 M_\odot$.

The duration of a microlensing event scales as $M^{\frac{1}{2}}$, and since for $\sim 10^{-1}M_\odot$, the observed

event durations are a few weeks, the shadow photino MACHOs should have microlensing event durations of order a few hours or days. Detection of such events by dedicated searches would support the interpretation suggested here as the origin of MACHOs in the required mass range. Although the details are model dependent, such U(1) factors in the hidden sector are generic in superstring theories and hence give rise to such CDM candidates.

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FIGURES

FIG. 1. M_{string} as a function of the hidden sector gauge group subject to the gaugino condensation condition. Solid lines correspond to $\mu_0 = 10^{13}$ GeV while short(long) dashed lines are for $\mu_0 = 0.4(4.0) \times 10^{13}$ GeV.

FIG. 2. $\alpha_s(M_Z)$ as a function of the quadratic Casimir for the highest rank hidden sector gauge group for an orbifold model with $\gamma = 3$ and taking $\mu_0 = 10^{13}$ GeV (solid curves). The upper and lower solid curves take into account the uncertainties in $\sin^2 \theta_W$ and α_{em}. The shifted short(long) dashed curves indicate the effect of changing μ_0 to $0.4(4.0) \times 10^{13}$ GeV.

FIG. 3. $\alpha_s(M_Z)$ as a function of the quadratic Casimir for the highest rank hidden sector gauge group for three values of γ (2.6,3.0,3.7) and taking $\mu_0 = 10^{13}$ GeV. The horizontal lines give the current LEP limits on $\alpha_s(M_Z)$ assuming the MSSM. The range $2.62 < \gamma < 3.73$ is that which just allows at least one U(1) component in the hidden sector gauge group. Uncertainties in $\sin^2 \theta_W$ and α_{em} are indicated by the shaded regions.